

ICS 2022 Problem Sheet #13

This sheet is only for students who failed to obtain the module achievement.

Problem 13.1: *sum formula* (2 points)

Prove that $1 + 4 + \dots + (3n - 2) = \frac{1}{2}n(3n - 1)$ for $n \in \mathbb{N}$ and $n > 0$.

Problem 13.2: *equivalence relation* (2 points)

Let $A = \mathbb{N}_+ \times \mathbb{N}_+$ be the set of pairs of positive natural numbers. Let $\sim \subseteq A \times A$ be a binary relation on A . The tuple $((a, b), (c, d))$ is an element of \sim if and only if $ad = bc$ (the product of a and d is equal to the product of b and c).

Show that \sim is an equivalence relation (i.e., \sim is reflexive, symmetric and transitive). For each property, first state what you are trying to show before you provide the argument.

Problem 13.3: *not-or is a universal boolean function* (2 points)

Prove that not-or (\downarrow) is a universal boolean function by showing and proving how \downarrow can be used to implement the Boolean functions \wedge, \vee, \neg .

Problem 13.4: *bnf grammar reduction* (2 points)

Let $\Sigma = \{0, 1, \dots, 9, x, y, z, +, *, (,)\}$. Consider the following grammar in Backus Naur Format:

```
<expression> ::= <term> | <expression> "+" <term>
<term>       ::= <factor> | <term> "*" <factor>
<factor>     ::= <constant> | <variable> | "(" <expression> ")"
<variable>  ::= "x" | "y" | "z"
<constant>  ::= <digit> | <digit> <constant>
<digit>     ::= "0" | "1" | "2" | "3" | "4" | "5" | "6" | "7" | "8" | "9"
```

- Use the grammar to reduce the expression $42 + 8 * x$ to the start symbol. Show each step of your derivation.
- Show four different examples of syntactically invalid expressions and describe which grammar rules are detecting the errors.

Problem 13.5: *divisors in haskell* (1 point)

Write a function `divisors :: Int -> [Int]` that returns the list of proper divisors of a given positive integer x . The result of `divisors x` includes 1, but not the number x itself. For example:

```
Prelude> divisors 6
[1,2,3]
Prelude> divisors 12
[1,2,3,4,6]
Prelude> divisors 15
[1,3,5]
Prelude> divisors 1
[]
Prelude> divisors 2
[1]
```

Recall that the Haskell function `div` gives you the result of an integer division (truncated toward negative infinity) and the function `mod` gives you the integer modulus (remainder of an integer division).

Problem 13.6: *folds in haskell*

(1 point)

Consider the following function definition:

```
1     m f xs = foldr g [] xs
2     where g y ys = (f y) : ys
```

How is the expression `m (*2) [1..3]` evaluated? Explain step-by-step how the expression is expanded and how the result is produced. Describe what the function `m` is doing, i.e., to which standard Haskell function it relates.