

### ICS 2022 Problem Sheet #3

**Problem 3.1:** *cartesian products*

(1+1 = 2 points)

Prove or disprove the following two propositions.

a)  $(A \cap B) \times (C \cap D) = (A \times C) \cap (B \times D)$

b)  $(A \cup B) \times (C \cup D) = (A \times C) \cup (B \times D)$

**Problem 3.2:** *reflexive, symmetric, transitive*

(3 points)

For each of the following relations, determine whether they are reflexive, symmetric, or transitive. Provide a reasoning.

- a) The absolute difference of the integer numbers  $a$  and  $b$  is less than or equal to 3.

$$R = \{ (a, b) \mid a, b \in \mathbb{Z} \wedge |a - b| \leq 3 \}$$

- b) The last digit of the decimal representation of the integer numbers  $a$  and  $b$  is the same.

$$R = \{ (a, b) \mid a, b \in \mathbb{Z} \wedge (a \bmod 10) = (b \bmod 10) \}$$

**Problem 3.3:** *total, injective, surjective, bijective functions*

(1+1 = 2 points)

Are the following functions total, injective, surjective, or bijective? Explain why or why not.

a)  $f : \mathbb{N} \mapsto \mathbb{N}$  with  $f(x) = 2x^2$

b)  $f : \mathbb{R} \mapsto \mathbb{R}$  with  $f(x) = x^2 + 6$

**Problem 3.4:** *types (haskell)*

(1+2 = 3 points)

- a) What is the type signature of the `zip` function? How many type variables appear in the type signature? Could it be more or less? Explain

- b) What are the types of the following expressions? Explain why!

$$2 + 3$$

$$2 + 9 \text{ `div` } 3$$

$$2 + 9 / 3$$

$$2 + \text{sqrt } 9$$