ICS 2020 Problem Sheet #6

Problem 6.1: *completeness of $\rightarrow$ and $\neg$* (2 points)

Prove that the two elementary boolean functions $\rightarrow$ (implication) and $\neg$ (negation) are universal, i.e., they are sufficient to express all possible boolean functions.

Problem 6.2: *boolean equivalence laws* (1+1+1+1+1 = 5 points)

Simplify the following Boolean formulas by repeatedly applying Boolean equivalence laws. (The simplified formulas contain at most one $\wedge$ or $\vee$.) Indicate in every step which law you have applied. You obtain points for the derivation, not for the result alone.

a) $\varphi(A, B) = (\neg A \vee \neg B) \wedge (\neg A \vee B) \wedge (A \vee \neg B)$

b) $\varphi(A, B, C) = (A \wedge \neg B) \vee (A \wedge \neg B \wedge C)$

c) $\varphi(A, B, C, D) = (A \vee \neg(B \wedge A)) \wedge (C \vee (D \vee C))$

d) $\varphi(A, B, C) = (\neg(A \wedge B) \vee \neg C) \wedge (\neg A \vee B \vee \neg C)$

e) $\varphi(A, B) = (A \vee B) \wedge (\neg A \vee B) \wedge (A \vee \neg B) \wedge (\neg A \vee \neg B)$

Problem 6.3: *conjunctive and disjunctive normal form* (2+1 = 3 points)

Consider the following boolean formula:

$$\varphi(P, Q, R, S) = (\neg P \vee Q) \wedge (\neg Q \vee R) \wedge (\neg R \vee S) \wedge (\neg S \vee P)$$

a) How many interpretations of the variables P, Q, R, S satisfy φ? Provide a proof for your answer (e.g., by providing a truth table).

b) Given the interpretations that satisfy φ, write the formula for φ in disjunctive normal form (DNF).