

### ICS Problem Sheet #3

**Problem 3.1:** *size of power sets (proof by induction)*

(3 points)

Proof the following statement by induction:

The number of elements in the power set  $\mathcal{P}(S)$  of a finite set  $S$  with  $n$  elements is  $2^n$ .

**Problem 3.2:** *reflexive, symmetric, transitive*

(1+1+1 = 3 points)

For each of the following relations, determine whether they are reflexive, symmetric, or transitive. Provide a reasoning.

- a)  $R = \{(a, b) \mid a, b \in \mathbb{Z} \wedge a \neq b\}$   
(The numbers  $a$  and  $b$  are different.)
- b)  $R = \{(a, b) \mid a, b \in \mathbb{Z} \wedge |a - b| \leq 3\}$   
(The absolute difference of the numbers  $a$  and  $b$  is less than or equal to 3.)
- c)  $R = \{(a, b) \mid a, b \in \mathbb{Z} \wedge (a \bmod 10) = (b \bmod 10)\}$   
(The last digit of the decimal representation of the numbers  $a$  and  $b$  is the same.)

**Problem 3.3:** *coprime (haskell)*

(1 point)

Two numbers are said to be coprime if their greatest common divisor is 1. Write a Haskell function (including its type signature) that determines whether two numbers are coprime. You can use the builtin function `gcd x y`, which returns the greatest (positive) integer that divides both  $x$  and  $y$ .

**Problem 3.4:** *fibonacci numbers (haskell)*

(1+1+1 = 3 points)

The Fibonacci sequence is a sequence of integers in which the first and second terms are both equal to 1 and each subsequent term is the sum of the two preceding it. The sequence starts as follows:

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, ...

The following code implements a tail recursive function `fib` in Haskell, which for a given integer  $n$  returns the  $n$ th Fibonacci number.

```
fib :: Integer -> Integer
fib n = tailFib n 1 1
    where tailFib 1 a b = a
          tailFib 2 a b = a
          tailFib n a b = tailFib (n-1) (a+b) a
```

Explain in detail how this tail recursive Haskell function works.

- a) What is the purpose of the first two lines?
- b) What is the purpose of the arguments of `tailFib`?
- c) How is `fib 6` calculated? Write down the recursive function calls.